RESEARCH BY OTHERS

The Second Open Letter to Ned Dewey, Published Posthumously,
On the Definitive Directions, Dimensions and Proportions of Our Solar–Planetary System:

BODE'S LAW EXPLAINED

BY WILLIAM ARNOLD

On April 2, 1976, Ned, you wrote me, "I now believe that the cause of cycles is connected with unit fractions of the orbital periods of the planets, but, of course, there are probably many other causes as well. I am convinced that the only way to provide viability to any scientific truth is publication in scientific journals.

"Incidentally, on March 29, the day that your letter was received, I started to reread *The Crack in the Cosmic Egg!* In your letter you speak of Bode's Law. Have you noticed that if we consider Neptune as a sport, or extra, Pluto fits into Bode's Law very nicely."

Introduction: In 1772 Johann Ebert Bode of Germany, the editor of *Astronomiches Jahrbuch*, an influential and prestigious European journal in astronomy, popularized by publication a little-known law of planetary spacing of bodies in space around the sun. It seemed to *predict* the possibility of other planets within our solar system. Since the very day when man first stood erect and looked skyward, the naked-eye planets were Mercury, Venus, Earth, Mars, Jupiter and Saturn.

But until the advent of the mathematical sequence, since known simply as "Bode's Law" of planetary distance, no one questioned seriously the absoluteness of 6 planets.

Basically, Bode argued for a planet between Mars and Jupiter. Kepler, one hunderd and fifty years earlier, had *predicted* a planet would be found there. In 1766 Titius, a mathematic's professor at the University of Wittenberg, discovered the law and Bode popularized it. Take a sequence of numbers, 0, 3, 6, 12, etc., add 4 to each term, making 4, 7, 10, 16, etc., then shift the decimal one place left, making .4, .7, 1, 1.6, etc., letting Mercury be .4, Venus .7, Earth 1 (the calculated distance from the sun to the Earth being the astronomer's standard unit, the so-called "astronomical" unit: a.u.), etc., and the resultant sequence of numbers approximates the calculated distances for the known planets.

Planet	Bode's Law	Calculated Distance
Mercury	0.4	0.39
Venus	0.7	0.72
Earth	1	1
Mars	1.6	1.52
?	2.8	?
Jupiter	5.2	5.2
Saturn	10.0	9.5
?	19.6	
?	38.8	
?	77.2	

In 1781 Herschel of England discovered in his telescope the planet Uranus beyond Saturn (calculated at 19.2) and this seemed to confirm Bode's Law, astronomers noted widely. The sky was divided into regions and active search begun for the planet between Mars and Jupiter.

In 1801 Piazzi of Italy discovered the asteroid Ceres between Mars and Jupiter as *predicted* by the law and astronomers hailed the event worldwide.

In 1846 Galle of Germany discovered the planet Neptune beyond Uranus, and yet because it was far from its *predicted* position the "law" seemed disproved (in science, the "exception" disproves the rule or law) and was rejected thereafter by astronomers.

In 1930 Tombaugh of America discovered the planet Pluto beyond Neptune but because it too was far from its *predicted* position "Bode's Law" fell into abject disrepute among astronomers.

Today the law of planetary distance is still widely rejected by astronomers--with a few brave exceptions.

The majority of astronomical critics seem unconvinced of the physical relevance of "Bode's Law." "It is not known whether the law has any physical significance,"

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Neptune	38.8	30.0
Pluto	77.2	39.5

published *Harper's Science Dictionary*. "J.D. Titius in 1766 proposed and J.E. Bode in 1772 popularized what is known in astronomy as the Titius-Bode Law, for which the physical basis is still not understood," published the *Encyclopedia Britannica*. "No theoretical basis for the rule has been generally accepted," published Fred Whipple in *Earth, Moon, and Planets*. "It is not known whether Bode's Law is of real significance, or whether it is due to pure chance," published Patrick Moore in *The A-Z of Astronomy*.

Other astronomical commentators are more optimistic. "Today some scientists believe that it may have a significance as yet unexplained," published Mattersdorf in *Insight Into Astronomy*. Weizsacker of Germany postulated a theory of a solar nebula vortex which caused Opik in *The Oscillating Universe* to publish, "It is interesting to note that the eddies or vortices provide an explanation also for Bode's Law." "Weizsacker has shown that these vortices can form stable configurations in groups of five around the central star only at definitive distances from the center in agreement with Bode's Law," published Motz in *This is Outer Space*. In an article titled "Beyond Pluto" published in *On Astronomy* Azimov investigated the possible existence of other planets using Bode's Law. "The more solid discovery of Bode and Titius, regarding the spacing of planets, harkened back to the idea of the ancient Greeks that the arrangement of the heavenly bodies was related to the spacing of notes in a musical chord," published Walter Sullivan in *We Are Not Alone*.

In summary, Guy Murchie, in *Music of the Spheres*, published, "Whether this curious if limping harmony among our sister spheres could possibly have derived from chance alone is a question that draws by no means as clear a concert of opinions."

In 1975 Pearce in *The Crack in the Cosmic Egg* raised the question anew for you and I, Ned. "Is there any physical relevance between Bode's Law and the calculated 'physical' distances between the sun and the planets?" In the second letter on the broad subject of our solar-planetary system and its effect upon the sunspot cycle and all other cycles, I will restrict myself to a detailed explanation of the physical relevance of Bode's Law. However, ancillary to our discussion, if there is a physical law relating distance, periodicity, speed and space as I believe there is as I outlined in my first letter *(Cycles*, Volume 28, Number

9, December, 1977), as observed by the formula $\pi d = t r$ (discussed at length in this letter), then it will add much to our knowledge of the solar-planetary system and will aid us in constructing a "physical" model.

Ned, you believe the cause of cycles is related to variations of planetary periodicity. You should know that periodicity (a significant "cycle" parameter, for sure), speed, distance, space, it's *all* the same thing, *numbers* if you observe Einstein's *principle of equivalency* and use the proper *transference equation*.

In order to explain "Bode's Law" I will have to digress into *A Concise History of Western Astronomy, its Physics and Math*.

Although modern astronomy originated in ancient Egypt (see first "certain" astronomically-calculated date: "July 16, 4226 BC" in Dr. Richard Parker's book *The Calendars of Ancient Egypt* and O. Neugebauer's book, *The Exact Sciences in Antiquity*, and jointly in *Ancient Egyptian Astronomical Texts*, volumes 1-4) I will restrict our brief survey of astronomy, physics and mathematics to its origins in Greece. In this sense, Pythagoras was responsible for the modern view that the order of the cosmos was fundamentally mathematical. He appears to have been the first western thinker to theorize a central sun about which all the planets, including earth, revolved. In essence, Pythagoras constructed the first physical model of the solar-planetary system.

In 590 BC Pythagoras was born on Samos and "thinking" has not been the same since. His primary discovery and teaching was mathematical physics. He acknowledged his own education included Egyptian astronomers (astrologers, priests or whatever tag-name one wishes upon them). He taught that "concordant intervals of the musical scale can be expressed in simple ratios: octave, 1:2, fourth, 4:3, and fifth, 3:2," according to Santillana in *The Origins of Thought*. Actually, the octave comes from doubling (2X) or halving (½X) the interval; the fourth comes from quartering (¼X or ¾X), according to Murchie. Not surprisingly, Pythagoras noted a strange similarity between musical intervals and the spacings of the planets.

His theory of numbers included 10 as the "perfect" number, being the sum of 1, 2, 3 and 4. His theory stated boldly: "All things are numbers." The number 1 expressed the point, 2 the line, 3 the triangle, and 4 the square. Strangely enough, the circle was not represented by a natural number but some "irrational" number between 3 and 4.

In 585 BC Thales of Miletus *predicted* a solar eclipse which brought him immense fame. This prediction of a rare event (Thales was reported to be the first western thinker to import Egyptian knowledge to Greece) proved ancient Greek astronomy was firmly based on physics and mathematics. Later masters, including Archimedes (287-212 BC), who trapped π between regular polygons with a decimal

equivalency of 3.141851 (Azimov in *On Numbers*), had firm knowledge of mathematics, including algebra, geometry, spherical geometry, etc. The 5-pointed star pentagon was the official sign of the Pythagoreans. The betrayal of Hippasos, a follower, was not by his revelation of the "irrational" numbers to outsiders but by his revelation of the construction of the dodecahedron, the sphere with 12 pentagons. In fact, Kepler constructed a "stellate" dodecahedron to better understand mathematically the solar-planetary system.

In 400 BC Hippolytos, another follower, said that Pythagoras "taught the Universe sings with the harmonious music of the spheres and that the motions of the seven heavenly bodies form a song." The seven known bodies were the Moon, Mercury, Venus, Mars, Jupiter, Saturn and the Sun or Earth. One of the latter was considered stationary. But the astronomical debate was much heated upon this central point.

The heliocentric, sun-centered, view of Pythagoras was an idea, a mathematical idea to be sure, and one which had to be mathematically proved. Obviously, the geocentric, earth-centered, view was the way most people saw heavenly things—the sun moving "around" the earth. We still say the sun "rises" and the sun "sets" and not that the earth "turns." Originally, a "stationary" sun and a "moving" earth was the idea of a physical mathematician—one who counted and measured things observed. As Guy Murchie put it in his masterful *Music of the Spheres*, Pythagoras "expressed a law of universal harmony under which the planets would be found to revolve in circles upon invisible spheres if only one could measure them in true prespective."

By 480 BC Parmenides of Elea was teaching that all things observed in the heavens was an ordered system. He taught that Venus was a planet ("wanderer" among the stars) and not an evening and morning star. He taught that there were 5 zones on the earth, the tropics and arctic circles and equatorial belt. He taught the sphericity of the earth (it cast a curved shadow at lunar eclipses) and that the moon shines by reflected sunlight. And in the true spirit of Pythagoras, he taught that "the different radii of planetary spheres must then have harmonic ratios; they become comparable to the lengths of the string, and the angular velocities to the frequencies of vibration," according to Santillana.

And by the late fourth century BC, Aristoxenos of Tarentum published *Elements of Harmony* in which "he defined the pitch difference between the fourth and fifth (intervals) as the unit tone of his scale," according to Murchie. "Aristoxenos would probably have been impressed by *the fact that all its tonal differences in frequency are multiples of the prime number 11*" (please note, Ned, and all scientific observers of the 11.11 year sunspot periodicity cycle, *italics* are mine). "And Pythagoras would surely applaud its plurality of means: E (5) the *arithmetic* mean between C (4) and G (6), F (5 and 1/3) the *harmonic* mean of C (4) and C' (8), and G (6) the *geometric* mean of

C (4) and D' (9)!" according to Murchie.

Thus, by 450 BC when Philolaos of Croton was teaching that the earth circled the sun, there was nothing new in this at all. In the same period Hippocrates of Chios was teaching his *Elements of Geometry*. Philolaos was teaching Archytas, who later taught Plato and Eudoxus and they, Aristotle. So when we learn that Archytas by 380 BC was teaching that pitch was determined by frequency, and observed publicly that each planet moved at different speed and distance, we should not be surprised by such "modern" ideas. Nor should we forget that it was Pythagoras' theory of conic sections as popularized by Apollonios of Perga which allowed Kepler and Newton to derive their respective laws about the interrelationships between planetary orbits, distances, periods, masses and gravitation.

Astronomer and solar-planetary system physicist and mathematician Philolaos was a devoted Pythagorean. He taught the "complete" model of the solar-planetary system. He taught that the sun was at the center of a massive sphere, liquid below and gas above, the planets *literally* floating on a solar sea in between the sun at center and the "hull" of the system. That is, *the system was not the center alone but included all the vast space out to the last planet* (Santillana, *Origins*, p. 69). Indeed, this is the *true* and *certain* heliocentric, sun-centered, solar-planetary system model being postulated today as a vast-rotating, electrically-induced, magnetically-aligned, solar "proton" sea and wind medium of interplanetary particles and waves by Dr. James A. Van Allen in "Interplanetary Particles and Fields" and Dr. Eugene N. Parker in "The Sun" in *Scientific American's Solar System* Book, 1975.

In 450 BC Empedocles of Acragas in *Physika* (a title he borrowed from Parmenides) not only taught that the solar-planetary system "operates in great cycles" and the earth turns on its axis in a daily cycle but that light has a "finite" speed—that is, a cycle length!

This true and certain science of astronomy with its heliocentric, sun-centered, solar-planetary system (having been observed and measured accurately and reported in cycles dating back 5,000 earth years) existed in the minds of intellectual Greeks and others of the Mediterranean culture for more than two hundred years during what is called the Golden Age. So what happened?

The problem as seen from the standpoint of most astronomers was not which theory best explained, but which best *predicted* the future positions of the orbiting bodies in space. Astronomers ("namers" of "stars") are really pragmatists at heart. Like the ancient Egyptian geometers, the men in the fields as the Nile flood receded who surveyed the land with lengths of rope, astronomers are as "down to earth" in accuracy when it comes to measuring things in the heavens.

Accordingly, the fact most militating against the sun-centered theory was varying size and brightness of the sun,

moon and planets, which seemed to indicate varying distances from the observer, according to Marshall Clagett in *Greek Science in Antiquity*. And how could there be "varying" distances inside "perfect" Pythagorean circles?

At the behest of Plato ("Save the appearance!"), the astronomer-physicist-mathematician Eudoxus published *On Speeds* and a complex theory of "eccentric" circles or epicycles which held the earth off-center at one focus. For observations he used a mechanical instrument equal to a modern transit. Thus, Eudoxus of Cnidus was reported to be the first to teach an eccentric version of the geocentric, earth-centered, system. He created accurate and complete star charts. And as a "mathematical" astronomer interested in physics, he chose to explain the varying brightness and size of observed bodies and the "irregular" movements with a seemingly "natural" system based upon the directly observed varying geocentric distances. The "perfect" circles of Pythagorean theory had left these anomalies unexplained. Eudoxus published mathematical proof that the "eccentric" circle was equal to the geometric figure of the deferent-epicycle. Later, Aristotle popularized this counter-Pythagorean theory in order to better *predict* the observed positions of the planets. And astronomers approved, able to find their planets with more accuracy in *prediction*.

Things changed slowly. Initially, when Plato, Eudoxus and Aristotle broke intellectually with the line of thinkers going back to Pythagoras, astronomy broke from astrology, physics from ethics, mechanics from motion, etc. Hence, departmentalization ("segmented" thinking) had come into being.

And yet by 280 BC Aristotle's popularization was not totally pervasive. In that period Aristarchos of Samos (homeport of Pythagoras) was still teaching that only the moon circled the earth, while all other observed planets, including the earth, circled the sun. He, too, taught that the earth rotated on its axis. So, the heliocentric, sun-centered, solar-planetary system was not yet dead.

In fact, Aristarchos' refinements to the Pythagorean "perfect" circle was the "perfect" center, the center of "magnitude" (which is so "modern" as to be the *very* subject of this letter, *post* Einstein, eventhough Archimedes doubted their existence). Aristarchos "had outlined the proper symmetry of homocentric circles and velocities, but it was clear already that they could not be true sun-centered circles. They were again *eccentrics with imaginary centers*," according to Santillana, *italics* mine.

As fate would have it (the cycle was to swing in favor of Aristotle) in 150 BC Hipparchos of Nicaea, an astronomer-mathematician, made new and broad observations of the heavens with better instruments and armed with "new" data he expanded upon the Aristotlian view, the geocentric. Hipparchos "adopted" the eccentric circle and center and taught the geocentric coordinate system of *positional* astronomy, "for which a whole technique had been developed and enriched by trigonometry," according to Santillana. In fact, it serves adequately earth-based astronomers today.

In 332 BC Alexander conquered Egypt and the Mediterranean. Student of Aristotle, he spread his master's word faithfully and most deeply at "Alexandria," Egypt. And for nearly 500 years the Aristotlian-Hipparchian geocentric, earth-centered, system ruled astronomy. In classic irony, the solar-planetary system of the ancient Egyptians and Greeks, particularly as popularized by the followers of Pythagoras, was finally *obliterated* from the minds of men and women and was literally and figuratively, *dead*.

And to make matters worse, in 150 AD, Claudius Ptolemy of Alexandria, last of the great Greek astronomers, popularized in his book *Mathematica Syntaxis* the geocentric view as was believed truth and certainty by "all" Alexandrians. Equipped with the latest trigonometric observational data, his system worked. And hence, today, the geocentric viewpoint is still synonymous with the "Ptolemaic" system.

So successful was this latest popularization with its accuracy of *predicting* planetary positions that it dominated western astronomical thinking for fifteen hundred years. Throughout what was known as the "dark" ages, the minds of men and women were literally in the dark and away from the light. Not until the 16th Century and the so-called "Copernican Revolution" did the heliocentric, sun-centered, solar-planetary system we know and talk about today come back into intellectual respect among western astronomical thinkers.

In 1543 AD Copernicus of Poland used Ptolemy's data in his now famous book *De Revolutionibus Orbium Coelestium* ("On the Revolutions of the Celestial Orbs") and presented a revived, heliocentric, sun-centered, solar-planetary system (quite neo-Pythagorean). But most of the book was in the well-established tradition of astronomical *prediction* and concerned itself with detailed analysis and tables of planetary motions. Owen Gingerich in "Copernicus and Tycho" (*Scientific American*, Dec. 1973) published, "Scarcely 20 pages of *De Revolutionibus* are devoted to the new heliocentric cosmology."

And although no thoughtful reader would consider Copernicus' thesis of a heliocentric system "new," events confirm what we have become aware of all along. Important with astronomers is *prediction* of planets, positions, etc. For explanations we need to look elsewhere. Important with solar-planetary system physicists is the *model*. And as you have reminded me again and again, Ned, in our letters and telephone conversations, we should not get lost in arguments about "details" on our way to a serious explication of the *cause* of cycles.

Important with cycle scientists is the knowledge that their fundamental principles will be found in the *theory* of probability statistics. The astronomers need to know, according to Santillana, that their fundamental principles

will be found in the *theory* of physics. Both need to know that fundamental to "all" *named* principles is the *theory* of mathematics. I emphasize the word *"theory"* in connection with "all" science theories so that none will be taken for granted as having cornered the market on *truth* and *certainty*.

Obviously, no current *model* of the solar-planetary system has found common acceptance among the various sciences mentioned. Hidden within the maze of words we "all" seek understanding. What *causes* cycles? Who among us would dare suggest cycles are circles? Or orbitals are waves? Do not astronomers accept the "eccentric" circle as "explanatory" of orbiting bodies? Is not the circle, the cycle, whether "perfect" or not, the indispensible tool of the modern astronomer-cycle scientist? Is not the sinusoidal cycle a length of waves? Astronomers need the physicist and the mathematician and the cycle scientist (probability theorist) to explain the "circular" orbits and "cyclical" events inherent in the system and the laws of numbers. "But modern computations of orbits are no less complicated, in fact more, *even if circles in vast numbers are inconspicuously tucked away in one line of print as convergent analytical series,"* according to Santillana, *italics* mine for emphasis.

Astronomers may be asking the "impossible" of system physicists. Heisenberg's *principle of uncertainty* postulates that if you measure for the "physical" particle then you cannot ascertain orbital or wave data, which is synonymous with saying that if you measure a planet's position you cannot ascertain *in that fact alone* its mean distance to the sun (unless, of course, you measure at the *nodes:* the intersection of the orbital plane of the planet with the solar ecliptic). This is the same dilemma of the study of light: is it "particular" or is it "waves"? Hence, the resolution to the age old problem of *describing* the model of our solar-planetary system "appears" within our grasp.

Probability theory postulates upon *an endless series of events* that if roughly 95% of all randomly recorded events are measured to be within ±2 standard deviations of the *mean*, or nodes, then we have in that mean an expectation of *high* probability that it predicts the mean wave of all events (note, however, that the results *cannot* be absolute in accordance to the principle of uncertainty). Astronomers are expecting 100% accuracy of *prediction* of planetary positions, as it is their right. But they are looking for the *particular* in one orbit, not the cyclic wave data inherent in an endless series of orbitals. In their zeal to be accurate and "measure the particle" they have as astronomers confused the physicist as to what to look for. Both have lost cognizance of the facts of the mean data (now the storehouse of the cycle scientist), its revelance to *description* of the wave shapes of the orbitals and hence, the *model* of the solar planetary system.

Obviously, cycle scientists as probability theorists are interested in the latter, the solar-planetary system model, and also in the 11.11 earth-year sunspot cycle and not the 11.86 earth-year particle (Jupiter) cycle. I hold to the thesis that what holds for the atom, holds for the solar-planetary system, relatively so. Guy Murchie's *Music of the Spheres* might well become our "bible," and the Rutherford-Bohr model of the atom might well become our starting point once again (but then Weizsacker's model, as well). I *predict* that the *model* of the solar-planetary system as outlined in the latter part of this letter, as viewed from a polar observer above the solar ecliptic, whether north or south, with its sequential orbitals, will match the wave shape of the diffraction patterns of Joseph Thomson, Ernest Rutherford and C.J. Davisson and that Louis deBroglie's wave theory and Erwin Schroedinger's equations with Einstein's modifications will match the patterns and the known entropy of the system will date significantly with the known longevity of the universe.

Hence, if as astronomical-physicists, and mathematical-cycle-scientists, we relax our scrutiny of the "particles" and concentrate instead upon the waves, we shall be well-launched upon a well-deserved study of the *cause* of cycles.

Ned, perhaps at this point in our disucssion we should bring forward your book, *Cycles, The Mysterious Forces That Trigger Events*, and remind ourselves of the numerous cycles and extensive data and *analytical facts* therein, which has triggered this entire conversation. You particularly cite Tycho Brahe's data and Johannes Kepler's laws as a source point for your particular quest: the cause of cycles?

In 1577 AD astronomer Tycho Brahe of Denmark tried to do precisely that. He used Copernicus' tables (based on Ptolemy's *Mathematica*) but found the planets were not where they were *predicted*. With sophisticated, better-calibrated, larger mechanical equipment, money from the King and armies of assistants and an observatory of his "own" not far from Hamlet's famous castle, Tycho, the last of the "naked-eye" astronomers, for twenty laborious years (1576-1596), created accurate star catalogs and made extensive measurements of the planets which were 30 times (30X) *more accurate* than Ptolemy's data. For example, without aid of a telescope (the telescope was not "perfected" until 1609 AD by Galileo), Tycho measured the year, the period of the earth round the sun—to *within one second!* And because the earth year does *not* vary more than ±40 milliseconds in any one year (see Whipple, p. 109), this was a *landmark* achievement!

But the new observational data of Tycho did not support the "perfect" circles implicit in the sun-centered system of Copernicus. Tycho, the astronomer, interested in predicting the planet (particle), rejected the wave theory of Copernicus and published support of Ptolemy, for which the data made more sense, in the interest of accuracy—the law of the astronomer. Tycho's data are impeccable although his solar-planetary theory was not.

In 1588 AD, according to Murchie, Johannes Kepler, one of Tycho's many assistants, read casually Tycho's description of a comet's "elliptical" orbit. A Pythagorean follower,

avowed astrologer with little interest in nightly vigils as an active astronomer, obviously bolstered by Copernicus' publishing breakthrough in a church-dominated Europe of heliocentric, sun-centered, solar-planetary system, Kepler held fast to his distant master's voice: *"All things are numbers!"* "Kepler was so steeped in astrology and mysticism that he was a virtual Pythagorean and never ceased looking for harmonies and unsuspected relationships between the motions of the several planets," according to Murchie. Kepler calculated his own conception as "16 May, AD 1571, at 4:37 am" and followed by "a pregnancy lasting 224 days, 9 hours and 53 minutes." Kepler was himself a master mathematician. So, in 1601 AD, after Tycho's death, it's not surprising that Kepler became Imperial Mathematician at the island of Hven observatory.

Remembering Aristarchos of Samos had modified the heliocentric view to a near "perfect" centered, near "perfect" circled solar-planetary system, Kepler remained

steadfast. He refused to believe that Tycho's data were less than accurate. He studied the "eccentric" circle and the "oval, egg-shaped" as well, and finally landed with both feet upon the mathematical "elliptical" circle within a neo-Pythagorean, Copernican system.

Between 1609-1618 AD Kepler published his now-famous three laws of planetary motion about a central sun. These laws satisfied *prediction* of planets for the astronomers even better than any published before (including Tycho-Ptolemy's), and Kepler's laws are recognized as pillars of celestial mechanics to this day (with Einsteinian relativity modifications, to be sure). These three laws are, simply (1) the planets move round in elliptical orbits, the sun at one focus, (2) a straight line bisecting the sun and the planet sweep out equal areas in equal time, and (3) the the square of the period is directly proportional to the cube of the radial distance separating them (see "equivalency" columns 3 and 4 of chart below).

Planet	(1) Mean Distance From Sun of Planet	(2) Period In Earth-Years	(3) Cube of Distance	(4) Square of Period
Mercury	0.387	0.241	0.058	0.058
Venus	0.723	0.615	0.378	0.378
Earth	1	1	1	1
Mars	1.524	1.881	3.538	3.540
Jupiter	5.203	11.862	140.707	140.851
Saturn	9.539	29.458	867.774	867.977

(Chart after Guy Murchie in *Music of the Spheres*, p. 76)

With the above data Kepler declared, noticing the non-orderly, non-Pythagorean gap between Mars and Jupiter, "I have become bolder, and now I place a planet between these two." History records, Kepler was the first man to *predict* a planetary orbital between Mars and Jupiter. Therefore, we need not be surprised when we discover that a century and a half later Titius of Wittenberg, a mathematic's professor at that German University, was publishing a law of the orderly spacing of the planets.

In 1766 AD Johann Daniel Titius of Germany an-

nounced a numerical sequence he had observed inherent in the data of the relative mean distance of the planets from the sun. He reasserted Kepler's *prediction* of a planetary orbital between Mars and Jupiter and, on his own and with Bode's aid, *predicted* the possibility of other planets beyond Saturn.

As cycle readers, Ned, I have brought us full circle, if you will allow me the Pythagorean pun. We began with Bode's Law and now we must continue—the delving into the mathematics therein.

The Math Sequence		The Additive Function: add 4	The Decimal Shift: divide by 10		The Math Resultant:
Mercury	0				.4
Venus	3	" 4	"	10	.7
Earth	6	" 4	"	10	1.0
Mars	12	" 4	"	10	1.6
Ceres	24	" 4	"	10	2.8
Jupiter	48	" 4	"	10	5.2
Saturn	96	" 4	"	10	10.0
Uranus	192	" 4	"	10	19.6
Neptune	384	" 4	"	10	38.8
Pluto	768	" 4	"	10	77.2

The 0 in the Titius-Bode sequence *cannot* mathematically be set to Mercury as the first representative body in our solar-planetary system, often referred to simply as

"our solar system." The 0 has to represent the sun, axiomatically, as the first member of that sequence of bodies in that it is the natural prime member, from which it

has no immediate predecessor; and, by the same mathematical law of the "order of the natural numbers N," Mercury as the prime natural body which has one and only one immediate predecessor must be represented by the prime natural number, 3.

By way of proof I will cite one modern standard *Encyclopedia of Mathematics* by Gellert, Kustner, Hellwich, Kastner, Editors, and Hirsch and Reichardt, Scientific Advisors, henceforth, the *EOM*: "Every natural number has exactly one immediate successor; for example, 96 is

successor of 95. This means that the sequence of natural numbers has no last member, it never breaks off. *The number 0 is not a successor; every natural number other than 0 has exactly one immediate predecessor; this means that the sequence of natural numbers has a beginning in its first member 0"* (page 20, italics mine).

Accordingly, the only natural and axiomatically mathematical sequence representation would be to set the sun to 0, Mercury to 3, Venus to 6, etc., as follows:

The Math Sequence

Sun	0
Mercury	3
Venus	6
Earth*	?
Mars	12
Ceres	24
Jupiter	48
Saturn	96
Uranus	192
Neptune*	?
Pluto	384

the fact earth is "questionable" would not "appear" to geocentric astronomer-mathematicians, to be sure.

the fact Neptune was *unknown* to Titius, obviously, did not have any bearing upon the original sequence.

Clearly observed, 9 out of 11 bodies of our solar-planetary system fit the mathematical sequence. The secondary manipulations of a 4 additive and decimal place shift are precisely that: *secondary* functions which obscured the truth inherent therein.

The editors of *EOM* caution all would-be mathematicians not to make this fundamental error whenever a correlation of natural things is to be made with natural numbers, thusly: "We begin with the natural numbers, together with the rules for the *elementary operations*, just as they present themselves as perfectly obvious to a naive person. But the *axiomatic build-up* follows immediately, starting from the natural numbers and leading up to the complex numbers.

"Even for these simple concepts a notation is used that was unknown to the Greeks and whose absence was one reason for an extremely cumbersome and unwieldly pres-

entation: the use of *letters* for numbers. Today it is taken for granted in schools. Here the notation is admirably suited to the basic mathematical concepts, but it is so easy to handle that sometimes there is the danger of thoughtless and mechanical manipulation of letters. This suggestive effect must be strongly opposed, especially in schools: the primary thing is the *mathematical idea*, and the computational working details are secondary—not the other way round."

"The primary thing is the *mathematical idea*."

By observation, then, we note that Earth and Neptune are not represented by the natural and axiomatical mathematical sequence 0, 3, 6, 12, doubling to infinity. However, if the entire sequence is looked at in reverse order, then it becomes apparent that Earth at 9 and Neptune at 288, respectively, are mathematically 3/4 of their successors 12 and 384.

The Math Sequence

Sun	0
Mercury	3
Venus	6
Earth	9*
Mars	12
Ceres	24
Jupiter	48
Saturn	96
Uranus	192
Neptune	288*
Pluto	384

9 and 288 are mathematically 3/4 of their successors, 12 and 384, respectively.

Now, clearly observed 11 out of 11 bodies of our solar-planetary system fit the two way sequence. The sun is back in the sequence, as well as Earth and Neptune—where

they belong. The sun is at 0, where it belongs in a sun-centered system. And all the other bodies are *suspended in space at naturally represented numbers.* Our secondary

mathematical manipulation is natural, as observed from a direction of OUT-IN (obviously, a quartering sequence), as is our primary mathematical sequence natural, as observed from a direction of IN-OUT (obviously, a doubling sequence).

The 4 additive and decimal shift by Titius in his original formulation of the mathematical sequence was developed to combine with the original 6 and create the sum 10 or 1 to represent the sun-earth radian. After the decimal shift, its value is 1, or unitary, and was necessary to represent the geocentric view of the so-called "astronomical" unit. In so doing, the sun was dropped from the sequence altogether in violation of sound mathematical technique. Each planetary orbital was thus shifted to new representative numbers in order to conform. This secondary mathematical manipulation thereby obscured the basic truth inherent in the mathematical sequence as originally developed.

The THREE UNIVERSAL LAWS OF SOLAR·PLANETARY SYSTEM DIRECTION are observed in this "new" fact: the solar center is in *truth* and *certainty* located "in" the solar-planetary system. And although the sun appears to "move" there, across the heavens, the sun is, *in fact, stationary* (with relative respect to the system). And although the sun appears to be "out" there, across the heavens, the sun is, *in fact, in* (with relative respect to the system). Hence the law recognizes 3 basic universal directions, (1) "up" and "down" describe *polar* movement (traditionally, perpendicular *to*) with respect to the solar-planetary ecliptic plane (the *mean:* in as much as the "nodes" are, *means*--see my paragraph on Heisenberg, previously; the cycle scientist, the probability theorist, starts with the fundamental cycle, circle, the calculus of nodal points of the solar-planetary system as observed over myriad cycles, revolutions, rotations, the *absolute* myriad locii describing *in an endless series of events* the "great" system circle, *the mathematical idea of* the "perfect" circle), "up" being *equated* with "north" and "down" being *equated* with "south," determined by the polar alignment of the solar-planetary system field (see essentials of electricity and magnetism in *Essentials of Electricity,* Slurzberg and Osterheld), and (2) "left" and "right" describe *rotational* movement about the polar axis of the solar-planetary system, "left" being *equated* with "north" and "right" being *equated* with "south," *indeterminate* of whether or not the observer looks up or down, and (3) "in" and "out" describe *planetary* movement "within" the solar-planetary system ecliptic plane, "in" being *equated* with "solar center" and "out" being *equated* with "planetary system."

Again, Ned, I wish to emphasize, these are the definitive laws of universal direction for all *known* physical and natural space. Inherent in Einstein's $E = mc^2$ was nuclear fission-fusion and by application nuclear power and the atom bomb, and inherent within my formula $\pi d = tr$ is spacial fission-fusion and by application spacial power and the so-called "antigravitational" machine. Knowledge for the cycle scientist includes the awareness that synonymous

with "in" is "tropical" time, "stationary" time, finite space and the "convergent" view, ♀, the convergent analytical series of mathematical computation. Knowledge for the astronomer includes the awareness that synonymous with "out" is "sidereal" time, "moving" time, *out*finite space and the "divergent" view, ♂, the divergent analytical series of mathematical computation. Both views, like both hemispheres of a sphere, *tautologically,* make a sphere – a "whole" *truth* and *certainty.*

Thus, round the solar-center are suspended the bodies in space in accordance to a mathematical sequence of *direct proportionality.* And Ned, does the cycle scientist, *really* appreciate the Pythagorean etymology inherent in "in a cord/chord dance"?

"Direct proportionality" is observed naturally, according to the *EOM* editors, in the simple action-reaction of a common spring. The more the suspended bodies, the greater is the extension of a "helical" spring. Thus, in a *certain* spring if a particular load of x units of bodies *causes* an extension of y units of length, they are said to be mathematically related in a *direct proportionality* which is ascertainable, measurable and relatable.

bodies (x)		lengths (y)	proportionality (c)
Sun	0	0	0
Mercury	1	3	3
Venus	2	6	3
Earth	3	9	3
Mars	4	12	3
Ceres	8	24	3
Jupiter	16	48	3
Saturn	32	96	3
Uranus	64	192	3
Neptune	96	288	3
Pluto	128	384	3

"In general," the *EOM* editors state, "two quantities x and y are said to be directly proportional if (1) to every value of one quantity there corresponds exactly one value of the second quantity and if (2) from every measure of x the appropriate measure y arises by multiplication by one and the same real number c.

"DIRECT PROPORTIONALITY $y = xc$ or $y/x = c$

"If this connection is represented in a rectangular coordinate system, the points (x,y) lie on a line through the origin. The number c is called *proportionality factor.* It characterizes the prevalent practical situation. In the example the spring constant (3) is characteristic for the spring used," according to the *EOM* editors, p. 37.

Obviously, Ned, the value for the solar spacial constant ("circa" 3?) lies within sight of our eyes but in as much as an explanation involves *physics* ("physical" mathematics *explains* the "relatables"), we should turn our attention directly to a "physical" explanation. And as the first step

in this "new" direction. we should make our solar-planetary system *model true* and *certain*. Hence, the geocentric, sun-earth, distance the so-called "astronomical" unit (a.u.), *cannot* be equated with the *unit* distance of our system. The unit distance is *equated* with the distance from the sun to the first natural body, assumed by observation to be, Mercury. The solar unitary distance (s.u.) *equals* 1.

Again, Ned, I wish to emphasize, that in order to be *true* and *certain* to the heliocentric, sun-centered, solar planetary system *model* as is the stated purpose of the physicist (and not necessarily the astronomer), the heliocentric *unit* must be programmed into the model. Thus, su(x) = 1.

"The realm of *Truth is* that of *mathematics* in its amplest formulation as our time has brought it forth: the domain of all the possibilities of rigorous thought," according to Santillana, *italics* mine.

"The primary thing is the mathematical idea. Truth is mathematics."

The unit length (y), 3, of the unit body (x), Mercury, is *equated* with the proportionality factor, 3. However, in as much as the perimeter of any unitary equilateral triangle is 2.6 and the perimeter of any unitary regular 6 sided polygon is 3 and the unitary regular 4 sided square is 4 and the "irrational" unitary circle is 3.14159265 etc., it "appears," Ned, that our *true* and *certain* "ideal" circles, cycles, or orbital proportionality factor is *equated* not with 3 but 3.14159265 etc.

The symbol for the solar constant proportionality factor is π. In 1557 AD Recorde of England first *equated* "two parallel lines" (=) in print with "equality," meaning "equal to," in mathematical notation. The equal sign seems adequate to our discussion. The essence is that we concentrate on the mathematical idea primarily, and not the letters which reflect that idea.
The use of d or delta (Δ) for diameter is adequate, as well. The use of these symbols π and d for the mathematical values of the proportionality factor, the perimeter and the diameter of a circle, cycle, sphere, whatever, has an interesting history best explained by Azimov in *On Numbers*:

"In the first place, what is π? Well, it is the *Greek* letter *pi* and it represents the ratio of the length of the perimeter of a circle to the length of its diameter. *Perimeter* is from the Greek *perimetron*, meaning "the measurement around," and *diameter* from the Greek *diametron*, meaning "the measurement through." For some obscure reason, while it is customary to use perimeter in the case of polygons, it is also customary to switch to the Latin *circumference* in speaking of circles. That is all right, I suppose (I am no purist), but it obscures the reason for the symbol π," according to Azimov.

The word "purist" is Pythagorean in origin and means in essence a "clean or clear" view to the *Truth*. Our point:

π is the *perimeter* of a *circle*, "the measurement around," and may vary in the measurement of natural things, whether trees or rocks or planets or stars or space, but never in the "ideal." Thereby, as cycle scientists we must project within our *model* the "great" circles of nodal points for all orbitals, and we can expect that π is 3.14159265 in these "ideal" circles or orbitals of the planets as plotted in an endless series of events in accordance with probability theory and that, *in fact*, this "circle" so-plotted is the *mean!*

Azimov continues, "Back about 1600 the English mathematician William Oughtred, in discussing the ratio of a circle's perimeter to its diameter, used the Greek letter π to symbolize the perimeter and the Greek letter d to symbolize the diameter. They were the first letters, respectively of *perimetron* and *diametron*.

"Now mathematicians often simplify matters by setting values *equal to unity* whenever they can," *italics mine again*, for emphasis. "For instance," Azimov continues, "they might talk of a circle of unit diameter. *In such a circle, the length of the perimeter is numerically equal to the ratio of perimeter to diameter.* (This is obvious to some of you, I suppose, and the rest of you can take my word for it.) *Since in a circle of unit diameter the perimeter equals the ratio, the ratio can be symbolized by π, the symbol of the perimeter.* And since circles of unit diameter are very frequently dealt with, the habit becomes quickly ingrained," accordingly, but *italics* mine again.

Azimov concludes, "The first top-flight man to use π as the symbol for the ratio of the length of a circle's perimeter to the length of its diameter was the Swiss mathematician Leonhard Euler, in 1737, and what was good enough for Euler was good enough for everyone else."

The unit length (y), 3.14159265 etc., of the unit body (x), Mercury, is *equated* with the proportionality factor or ratio or orbital, 3.14159265 etc. The mathematical sequence derivative from my formula is as follows:

bodies (d)		perimeter (π)	proportionality (π) factor or ratio
Sun	0°	0°	0°
Mercury	1	1π	π
Venus	2	2π	π
Earth	3	3π	π
Mars	4	4π	π
Ceres	8	8π	π
Jupiter	15	15π	π
Saturn	30	30π	π
Uranus	60	60π	π
Neptune	90	90π	π
Pluto	120	120π	π

Although the sun (0°) is mathematically set to 0 in as much as that is its true dimension and proportionality status within the solar-planetary system, the photosphere of the sun, its diameter and perimeter, are *equally* so related and I wish to make note of the *fact* now, but I

bodies	Arnold's Law (ideal wave)	
Sun	0	
Mercury	3.14	$\times 10^7$ miles
Venus	6.28	"
Earth	9.42	"
Mars	12.56	"
Ceres	25.13	"
Jupiter	47.12	"
Saturn	94.24	"
Uranus	188.49	"
Neptune	282.74	"
Pluto	376.99	"

Obviously, our solar-planetary system of 10 orbitals round a central star is "ideally" arc'd proportionally, Pythagorean in its "assumed" completeness, although I cannot dismiss the possibility of other orbiting bodies nor our proximity and connection to other systems (more on this point later). Either our system is "unique" to our solar-planetary system spacial constant (π), or it *is*, *in fact*, universal. I am very inclined to the belief that "all" space is spherical. This follows naturally for me, in that I am theoretical in nature, and natural in inclination (see also Carl Heiles' "elliptical or circular feature of the shell structure of the interstellar medium," *Scientific American*, Jan 78. Also note the orderly mathematical sequence between circles, cycles, spheres, bubbles, hulls, shells—what I call "systems," and related electrical and magnetic orientations: polar, rotational and planetary).

The value for π in my above calculations is certainly an approximation. The editors of *EOM* caution would-be mathematical-astronomer-physicists accordingly. "This number is one of the most important and interesting mathematical constants. One can find arbitrary accurate approximations of π by increasing the number of sides of the polygons used. Archimedes used a 96-gon and found bounds that are still frequently used today. His values are

$$ 3^{10}/71 < \pi < 3^{10}/70 \text{ or } 3.14084507 < \pi < 3.14285714. $$

shall have to wait until another letter, Ned, for elaboration.

From the above mathematical sequence as "discovered" within the data we derive the following "law":

(ideal) proportions	(ideal) degreed arcs°
0	0°
1/120	3°
1/60	6°
1/40	9°
1/30	12°
1/15	24°
1/8	45°
1/4	90°
1/2	180°
3/4	270°
4/4	360°

The first 40 places after the decimal point are given by

"π= 3.14159 26535 89793 23846 26433 83279 50288 41971...

"The following rough calculation shows what an accuracy of 'only' 30 places of decimal means. A system of stars that astronomers can just make visible by hour-long exposures on photographic plates using the most powerful telescope, emitted the light that is trapped by the plate about 2000 million years ago. Since light travels about 9.5×10^{12} km per year, these stars are about $2 \times 10^9 \times 9.5 \times 10^{12}$ km = 1.9×10^{22} km away from the earth. The perimeter of a circle with this enormous distance as radius is

$\pi = 3.14$ etc. d = 3.8×3.14 etc. $\times 10^{22}$ km.

If in calculating this perimeter of a circle only the first 30 decimal places of π are used, the error occurs in the eighth place after the decimal point and is of the order of about 2 units. That is, the error caused by disregarding further places of π is about 20 micrometers or 0.02mm," according to the *EOM* editors.

Hence, if my law is compared with the calculated "mean" distances, the variations therefrom may say much about the solar-planetary system state-of-being versus the "ideal" model. Such follows:

bodies	Calculated Means**		less/more	Arnold's Law	
Sun	0			0	
Mercury	3.64×10^7 miles		>	3.14 etc. $\times 10^7$ miles	
Venus	6.77	"	>	6.28	"
Earth	9.34	"	<	9.42	"
Mars	14.21	"	>	12.56	"
Ceres*	25.13	"	=	25.13	"
Jupiter	48.43	"	>	47.12	"
Saturn	88.85	"	<	94.24	"
Uranus	178.00	"	<	188.49	"
Neptune	279.15	"	<	282.74	"
Pluto	365.43	"	<	376.99	"

*Ceres: prime representative of so-called "asteroids"

**means, adjusted for diameters of both bodies.

Obviously, significant and detailed study of the variations of the calculated mean distances from the "law" (as observed above), especially in the case of Mercury, Venus, and Mars "tending toward" Jupiter and Jupiter "tending toward" and being "tended toward" by Saturn, Uranus, Neptune and Pluto, the *massive* giants, will explain much about the current entropy of the solar-planetary system. Noting this, no one should "wonder" what happened to Ceres. Note the exception: Earth. (For the detailed understanding of "entropy" and the second law of thermodynamics, see Mendelssohn, *The Quest for Absolute Zero, the Meaning of Low Temperature Physics.*)

Equally significant and detailed study of the variations of the calculated ranges of distance from the "law" (as observed below), also, will explain much. Such follows:

bodies	Calculated Ranges**			less/more	Arnold's Law
Sun	0				0
Mercury	2.90X10⁷miles	-- --	-4.38X10⁷miles	=	3.14etc.X10⁷miles
Venus	6.72 "	--	6.82 "	>	6.28 "
Earth	9.19X "	--	9.50 "	=	9.42 "
Mars	12.89 "	--	15.54 "	>	12.56 "
Ceres*	18.60 "	--	37.30 "	=	25.13 "
Jupiter	46.11 "	--	50.75 "	≈	47.12 "
Saturn	83.89 "	--	93.80 "	<	94.24 "
Uranus	169.98 "	--	186.02 "	<	188.49 "
Neptune	276.08 "	--	282.21 "	<	282.74 "
Pluto	275.69 "	--	455.18 "	=	376.99 "

*Ceres: prime representative of so-called "asteroids."

**ranges, adjusted for diameters of both bodies.

Clearly observed 11 out of 11 bodies of our solar-planetary system fit the two way sequence model. The sun is back in the mathematical sequence, as well as Earth and Neptune — where they belong. The sun is at 0, where it belongs in a sun-centered system. And all the other bodies are *suspended in space* at naturally represented numbers, 0, 1, 2, 3, 4, 8, 15, 30, 60, 90, and 120. And the specific variations, Ned, in response to your *opening* quote to this letter, from the "law" (as observed above and in the case with the means), naturally support Newton's gravitational attraction principles for the planets (including Ceres and the "planetoids"), particularly the outer *massive* ones, the major causes of the major cycles. It will be shown later that *the G constant is a function of the π constant!*

In summary, then, Ned, astronomer-physicists can now recognize the "law" of the relative placing of the planets in space, the so-called planetary distance law, inherent within $\pi d = tr$, that is, $d = tr/\pi$, and apply it fully to the solar-planetary system model under development here. This formula is derived from *equating* the perimeter of the unitary circle ($\pi = 3.14etc.d$) with the distance travelled by an orbiting body ($D = tr$). Thus, $\pi d = tr$. This elastic space ($\pi = tr/d$) and elastic time ($t = \pi d/r$) and elastic speed ($r = \pi d/t$) and elastic distance ($d = tr/\pi$) formula is *the* space-time-speed distance continuum formula so necessary to correlating the "whole" system to its constituent parts and particularly the "field" and which allows Einstein's *principle of equivalency* to transfer us from one "system" to another *equally.*

Sincerely yours,

Edgartown, Mass.

Bill Arnold